

# Elementary particles in the early Universe

N. A. Gromov

Komi Science Center of the Ural Division  
of the Russian Academy of Sciences, Syktyvkar, 167982, Russia.  
Electronic address: gromov@dm.komisc.ru

## Abstract

The high-temperature limit of Standard Model generated by the contractions of gauge groups is discussed. Contraction parameters of gauge group  $SU(2)$  of Electroweak Model and gauge group  $SU(3)$  of Quantum Chromodynamics are taken identical and tending to zero when temperature increase. Properties of the elementary particles change drastically at the infinite temperature limit: all particles lose masses, all quarks are monochromatic. Electroweak interactions become long-range and are mediated by the neutral currents. Particles of different kind do not interact. It looks like some stratification with only one sort of particles in each stratum. The Standard Model passes in this limit through several stages, which are distinguished by the powers of contraction parameter. For any stage the intermediate models are constructed and the exact expressions for the respective Lagrangians are presented. The developed approach describes the evolution of Standard Model in the early Universe from the Big Bang up to the end of several nanoseconds.

## 1 Introduction

Modern theory of elementary particles known as Standard Model (SM) consist of Electroweak Model (EWM), which unified electromagnetic and weak interactions, as well as Quantum Chromodynamics (QCD), describing their strong interactions. Standard Model gives a good description of the experimental data and was recently confirmed by discovering of Higgs boson at LHC. Standard Model is a gauge theory with  $SU(3) \times SU(2) \times U(1)$  gauge group, which is direct product of a simple groups. The operation of group contraction (or limit transition) [1] well known in physics transforms a simple group to a non-semisimple one. For a symmetric physical system the contraction of its symmetry group means a transition to some limit state. In the case of a complicated physical system the investigation of its limit states under the limit values of some of its parameters enables to better understand the system behavior.

For the modified Electroweak Model with the contracted gauge group  $SU(2; j) \times U(1)$  it was demonstrated [2, 3, 4], that the contraction parameter is connected with system energy, so its zero limit corresponds to the low-energy limit of Electroweak Model. The alternative rescaling of the gauge group and the field space gives the high-temperature limit of Electroweak Model.

In the broad sense of the word deformation is an operation inverse to contraction. The non-trivial deformation of some algebraic structure generally means its non-evident

generalization. Quantum groups [5], which are simultaneously non-commutative and non-cocommutative Hopf algebras, present a good example of similar generalization since previously Hopf algebras with only one of these properties were known. But when the contraction of some mathematical or physical structure is performed one can reconstruct the initial structure by the deformation in the narrow sense moving back along the contraction way.

We use this method in order to re-establish the evolution of elementary particles and their interactions in the early Universe. We are based on the modern knowledge of the particle world which is concentrated in Standard Model. In this paper we investigate the high-temperature limit of Standard Model generated by contraction of the gauge groups  $SU(2)$  and  $SU(3)$ . Similar very high temperatures can exist in the early Universe after inflation and reheating on the first stages of the Hot Big Bang [6]. At these times the elementary particles demonstrate rather unusual properties. It appears that the SM Lagrangian falls to a number of terms which are distinguished by the powers of the contraction parameter  $\epsilon \rightarrow 0$ . As far as the temperature in the hot Universe is connected with its age, then moving forward in time, i.e. back to high-temperature contraction, we conclude that after the Universe creation elementary particles and their interactions pass a number of stages in their evolution from infinite temperature state up to SM state. These stages of quark-gluon plasma formation and color symmetries restoration are distinguished by the powers of contraction parameter and consequently by times of its creations. From the contraction of Standard Model we can classify the stages in time as earlier-later, but we can not determine their absolute date. To estimate the absolute date we use additional assumptions.

## 2 High-Temperature Lagrangian of EWM

It was shown in papers [2, 3, 4], that in zero energy limit the gauge group  $SU(2; j)$  and the fundamental representation space  $\mathbf{C}_2(j)$  are transformed as follows

$$z'(j) = \begin{pmatrix} jz'_1 \\ z'_2 \end{pmatrix} = \begin{pmatrix} \alpha & j\beta \\ -j\bar{\beta} & \bar{\alpha} \end{pmatrix} \begin{pmatrix} jz_1 \\ z_2 \end{pmatrix} = u(j)z(j), \quad z(j) = \begin{pmatrix} j\nu_l \\ e_l \end{pmatrix}, \dots, \begin{pmatrix} ju_l \\ d_l \end{pmatrix}, \dots, \\ \det u(j) = |\alpha|^2 + j^2|\beta|^2 = 1, \quad u(j)u^\dagger(j) = 1, \quad (1)$$

so for  $j \rightarrow 0$  the first components of the lepton and quark doublets become infinitely small in comparison to their second components. On the contrary, when energy (temperature) increases the first components of the doublets become greater than their second ones. In the infinite temperature limit the second components of the lepton and quark doublets will be infinitely small as compared to their first components. To describe this limit we introduce [7, 8] *new contraction parameter*  $\epsilon$  and *new consistent rescaling* of the group  $SU(2; \epsilon)$  and the space  $\mathbf{C}_2(\epsilon)$  in the form

$$z'(\epsilon) = \begin{pmatrix} z'_1 \\ \epsilon z'_2 \end{pmatrix} = \begin{pmatrix} \alpha & \epsilon\beta \\ -\epsilon\bar{\beta} & \bar{\alpha} \end{pmatrix} \begin{pmatrix} z_1 \\ \epsilon z_2 \end{pmatrix} = u(\epsilon)z(\epsilon), \\ \det u(\epsilon) = |\alpha|^2 + \epsilon^2|\beta|^2 = 1, \quad u(\epsilon)u^\dagger(\epsilon) = 1. \quad (2)$$

Hermite form

$$z^\dagger(\epsilon)z(\epsilon) = |z_1|^2 + \epsilon^2|z_2|^2 \quad (3)$$

remain invariant under contraction  $\epsilon \rightarrow 0$ .

In the contraction scheme (2) the standard boson fields and left lepton and quark fields are transformed as follows

$$\begin{aligned} W_\mu^\pm &\rightarrow \epsilon W_\mu^\pm, \quad Z_\mu \rightarrow Z_\mu, \quad A_\mu \rightarrow A_\mu. \\ e_l &\rightarrow \epsilon e_l, \quad d_l \rightarrow \epsilon d_l, \quad \nu_l \rightarrow \nu_l, \quad u_l \rightarrow u_l. \end{aligned} \quad (4)$$

The next reason for inequality of the first and second doublet components is the special mechanism of spontaneous symmetry breaking, which is used to generate mass of vector bosons and other elementary particles of the model. In this mechanism one of Lagrangian ground states  $\phi^{vac} = \begin{pmatrix} 0 \\ v \end{pmatrix}$  is taken as vacuum of the model and then small field excitations  $v + \chi(x)$  with respect to this vacuum are regarded. So Higgs boson field  $\chi$  and constant  $v$  are multiplied by  $\epsilon$ . As far as masses of all particles are proportionate to  $v$  we obtain the following transformation rule

$$\chi \rightarrow \epsilon \chi, \quad v \rightarrow \epsilon v, \quad m_p \rightarrow \epsilon m_p, \quad p = \chi, W, Z, e, u, d. \quad (5)$$

After transformations (4), (5) the EWM boson Lagrangian [9, 10] can be represented in the form

$$L_B(\epsilon) = -\frac{1}{4}\mathcal{Z}_{\mu\nu}^2 - \frac{1}{4}\mathcal{F}_{\mu\nu}^2 + \epsilon^2 L_{B,2} + \epsilon^3 g W_\mu^+ W_\mu^- \chi + \epsilon^4 L_{B,4}, \quad (6)$$

where

$$\begin{aligned} L_{B,4} &= m_W^2 W_\mu^+ W_\mu^- - \frac{1}{2} m_\chi^2 \chi^2 - \lambda v \chi^3 - \frac{\lambda}{4} \chi^4 + \frac{g^2}{4} (W_\mu^+ W_\nu^- - W_\mu^- W_\nu^+)^2 + \frac{g^2}{4} W_\mu^+ W_\nu^- \chi^2, \\ L_{B,2} &= \frac{1}{2} (\partial_\mu \chi)^2 + \frac{1}{2} m_Z^2 (Z_\mu)^2 - \frac{1}{2} \mathcal{W}_{\mu\nu}^+ \mathcal{W}_{\mu\nu}^- + \frac{gm_z}{2 \cos \theta_W} (Z_\mu)^2 \chi + \frac{g^2}{8 \cos^2 \theta_W} (Z_\mu)^2 \chi^2 - \\ &\quad - 2ig (W_\mu^+ W_\nu^- - W_\mu^- W_\nu^+) (\mathcal{F}_{\mu\nu} \sin \theta_W + \mathcal{Z}_{\mu\nu} \cos \theta_W) - \\ &\quad - \frac{i}{2} e \left[ A_\mu (\mathcal{W}_{\mu\nu}^+ W_\nu^- - \mathcal{W}_{\mu\nu}^- W_\nu^+) + \frac{i}{2} e A_\nu (\mathcal{W}_{\mu\nu}^+ W_\mu^- - \mathcal{W}_{\mu\nu}^- W_\mu^+) \right] - \\ &\quad - \frac{i}{2} g \cos \theta_W \left[ Z_\mu (\mathcal{W}_{\mu\nu}^+ W_\nu^- - \mathcal{W}_{\mu\nu}^- W_\nu^+) - \right. \\ &\quad \left. - Z_\nu (\mathcal{W}_{\mu\nu}^+ W_\mu^- - \mathcal{W}_{\mu\nu}^- W_\mu^+) \right] - \frac{e^2}{4} \left\{ \left[ (W_\mu^+)^2 + (W_\mu^-)^2 \right] (A_\nu)^2 - \right. \\ &\quad \left. - 2 (W_\mu^+ W_\nu^+ + W_\mu^- W_\nu^-) A_\mu A_\nu + \left[ (W_\nu^+)^2 + (W_\nu^-)^2 \right] (A_\mu)^2 \right\} - \\ &\quad - \frac{g^2}{4} \cos \theta_W \left\{ \left[ (W_\mu^+)^2 + (W_\mu^-)^2 \right] (Z_\nu)^2 - \right. \\ &\quad \left. - 2 (W_\mu^+ W_\nu^+ + W_\mu^- W_\nu^-) Z_\mu Z_\nu + \left[ (W_\nu^+)^2 + (W_\nu^-)^2 \right] (Z_\mu)^2 \right\} - \end{aligned} \quad (7)$$

$$-eg \cos \theta_W \left[ W_\mu^+ W_\mu^- A_\nu Z_\nu + W_\nu^+ W_\nu^- A_\mu Z_\mu - \frac{1}{2} (W_\mu^+ W_\nu^- + W_\nu^+ W_\mu^-) (A_\mu Z_\nu + A_\nu Z_\mu) \right]. \quad (8)$$

The lepton Lagrangian in terms of electron and neutrino fields takes the form

$$\begin{aligned} L_L(\epsilon) &= L_{L,0} + \epsilon^2 L_{L,2} = \\ &= \nu_l^\dagger i \tilde{\tau}_\mu \partial_\mu \nu_l + e_r^\dagger i \tau_\mu \partial_\mu e_r + g' \sin \theta_w e_r^\dagger \tau_\mu Z_\mu e_r - g' \cos \theta_w e_r^\dagger \tau_\mu A_\mu e_r + \frac{g}{2 \cos \theta_w} \nu_l^\dagger \tilde{\tau}_\mu Z_\mu \nu_l + \\ &\quad + \epsilon^2 \left\{ e_l^\dagger i \tilde{\tau}_\mu \partial_\mu e_l - m_e (e_r^\dagger e_l + e_l^\dagger e_r) + \frac{g \cos 2\theta_w}{2 \cos \theta_w} e_l^\dagger \tilde{\tau}_\mu Z_\mu e_l - \right. \\ &\quad \left. - e e_l^\dagger \tilde{\tau}_\mu A_\mu e_l + \frac{g}{\sqrt{2}} (\nu_l^\dagger \tilde{\tau}_\mu W_\mu^+ e_l + e_l^\dagger \tilde{\tau}_\mu W_\mu^- \nu_l) \right\}. \end{aligned} \quad (9)$$

The quark Lagrangian in terms of u- and d-quarks fields can be written as

$$L_Q(\epsilon) = L_{Q,0} - \epsilon m_u (u_r^\dagger u_l + u_l^\dagger u_r) + \epsilon^2 L_{Q,2}, \quad (10)$$

where

$$\begin{aligned} L_{Q,0} &= d_r^\dagger i \tau_\mu \partial_\mu d_r + u_l^\dagger i \tilde{\tau}_\mu \partial_\mu u_l + u_r^\dagger i \tau_\mu \partial_\mu u_r - \\ &\quad - \frac{1}{3} g' \cos \theta_w d_r^\dagger \tau_\mu A_\mu d_r + \frac{1}{3} g' \sin \theta_w d_r^\dagger \tau_\mu Z_\mu d_r + \frac{2e}{3} u_l^\dagger \tilde{\tau}_\mu A_\mu u_l + \\ &\quad + \frac{g}{\cos \theta_w} \left( \frac{1}{2} - \frac{2}{3} \sin^2 \theta_w \right) u_l^\dagger \tilde{\tau}_\mu Z_\mu u_l + \frac{2}{3} g' \cos \theta_w u_r^\dagger \tau_\mu A_\mu u_r - \frac{2}{3} g' \sin \theta_w u_r^\dagger \tau_\mu Z_\mu u_r, \end{aligned} \quad (11)$$

$$\begin{aligned} L_{Q,2} &= d_l^\dagger i \tilde{\tau}_\mu \partial_\mu d_l - m_d (d_r^\dagger d_l + d_l^\dagger d_r) - \frac{e}{3} d_l^\dagger \tilde{\tau}_\mu A_\mu d_l - \\ &\quad - \frac{g}{\cos \theta_w} \left( \frac{1}{2} - \frac{2}{3} \sin^2 \theta_w \right) d_l^\dagger \tilde{\tau}_\mu Z_\mu d_l + \frac{g}{\sqrt{2}} [u_l^\dagger \tilde{\tau}_\mu W_\mu^+ d_l + d_l^\dagger \tilde{\tau}_\mu W_\mu^- u_l]. \end{aligned} \quad (12)$$

The complete Lagrangian of the modified model is given by the sum  $L(\epsilon) = L_B(\epsilon) + L_L(\epsilon) + L_Q(\epsilon)$  and can be written in the form

$$L(\epsilon) = L_\infty + \epsilon L_1 + \epsilon^2 L_2 + \epsilon^3 L_3 + \epsilon^4 L_4. \quad (13)$$

The contraction parameter is monotonous function  $\epsilon(T)$  of the temperature with the property  $\epsilon(T) \rightarrow 0$  for  $T \rightarrow \infty$ . Very high energies can exist in the early Universe just after its creation.

It is well known that to gain a better understanding of a physical system it is useful to investigate its properties for limiting values of physical parameters. It follows from the decomposition (13) that there are five stages in evolution of the Electroweak Model after the creation of the Universe which are distinguished by the powers of the contraction parameter  $\epsilon$ . This offers an opportunity for construction of intermediate limit models. One can take the Lagrangian  $L_\infty$  for the initial limit system, then add  $L_1$  and obtain the second limit model with the Lagrangian  $\mathcal{L}_1 = L_\infty + L_1$ . After that one can add  $L_2$  and obtain the third limit model  $\mathcal{L}_2 = L_\infty + L_1 + L_2$ . The last limit model has the Lagrangian  $\mathcal{L}_3 = L_\infty + L_1 + L_2 + L_3$ .

In the infinite temperature limit ( $\epsilon = 0$ ) Lagrangian (13) is equal to

$$L_\infty = -\frac{1}{4} \mathcal{Z}_{\mu\nu}^2 - \frac{1}{4} \mathcal{F}_{\mu\nu}^2 + \nu_l^\dagger i \tilde{\tau}_\mu \partial_\mu \nu_l + u_l^\dagger i \tilde{\tau}_\mu \partial_\mu u_l +$$

$$+ e_r^\dagger i \tau_\mu \partial_\mu e_r + d_r^\dagger i \tau_\mu \partial_\mu d_r + u_r^\dagger i \tau_\mu \partial_\mu u_r + L_\infty^{int}(A_\mu, Z_\mu), \quad (14)$$

where

$$\begin{aligned} L_\infty^{int}(A_\mu, Z_\mu) = & \frac{g}{2 \cos \theta_w} \nu_l^\dagger \tilde{\tau}_\mu Z_\mu \nu_l + \frac{2e}{3} u_l^\dagger \tilde{\tau}_\mu A_\mu u_l + \\ & + g' \sin \theta_w e_r^\dagger \tau_\mu Z_\mu e_r + \frac{g}{\cos \theta_w} \left( \frac{1}{2} - \frac{2}{3} \sin^2 \theta_w \right) u_l^\dagger \tilde{\tau}_\mu Z_\mu u_l - g' \cos \theta_w e_r^\dagger \tau_\mu A_\mu e_r - \\ & - \frac{1}{3} g' \cos \theta_w d_r^\dagger \tau_\mu A_\mu d_r + \frac{1}{3} g' \sin \theta_w d_r^\dagger \tau_\mu Z_\mu d_r + \frac{2}{3} g' \cos \theta_w u_r^\dagger \tau_\mu A_\mu u_r - \frac{2}{3} g' \sin \theta_w u_r^\dagger \tau_\mu Z_\mu u_r. \end{aligned} \quad (15)$$

We can conclude that the limit model includes only *massless particles*: photons  $A_\mu$  and neutral bosons  $Z_\mu$ , left quarks  $u_l$  and neutrinos  $\nu_l$ , right electrons  $e_r$  and quarks  $u_r, d_r$ . This phenomenon has simple physical explanation: the temperature is so high, that particle mass becomes negligible quantity as compared to its kinetic energy. The electroweak interactions become long-range because they are mediated by the massless neutral  $Z$ -bosons and photons. Let us note that  $W_\mu^\pm$ -boson fields, which correspond to the translation subgroup of Euclid group  $E(2)$ , are absent in the limit Lagrangian  $L_\infty$  (14).

Similar high energies can exist in early Universe after inflation and reheating on the first stages of the Hot Big Bang [6, 11] in pre-electroweak epoch. However more interesting is the Universe evolution and the limit Lagrangian  $L_\infty$  can be considered as a good approximation near the Big Bang just as the nonrelativistic mechanics is a good approximation of the relativistic one at low velocities.

From the explicit form of the interaction part  $L_\infty^{int}(A_\mu, Z_\mu)$  it follows that there are no interactions between particles of different kind, for example neutrinos interact only with each other by neutral currents. All other particles are charged and interact with particles of the same sort by massless  $Z_\mu$ -bosons and photons. Particles of different kind do not interact. It looks like some stratification of the Electroweak Model with only one sort of particles in each stratum.

From contraction of the Electroweak Model we can classify events in time as earlier-later, but we can not determine their absolute time without additional assumptions. Already at the level of classical gauge fields we can conclude that the  $u$ -quark first restores its mass in the evolution of the Universe. Indeed the mass term of  $u$ -quark in the Lagrangian (13)  $L_1 = -m_u(u_r^\dagger u_l + u_l^\dagger u_r)$  is proportional to the first power  $\epsilon$ , whereas the mass terms of  $Z$ -boson, electron and  $d$ -quark are multiplied by the second power of the contraction parameter

$$\epsilon^2 \left[ \frac{1}{2} m_Z^2 (Z_\mu)^2 + m_e(e_r^\dagger e_l + e_l^\dagger e_r) + m_d(d_r^\dagger d_l + d_l^\dagger d_r) \right]. \quad (16)$$

At the same time massless Higgs boson  $\chi$  and charged  $W$ -boson are appeared. They restore their masses after all other particles of the Electroweak Model because their mass terms are multiplied by  $\epsilon^4$ .

The electroweak interactions between elementary particles are restored mainly in the epoch which corresponds to the second order of the contraction parameter. There is one term in Lagrangian (6)  $L_3 = g W_\mu^+ W_\mu^- \chi$  proportionate to  $\epsilon^3$ . The final reconstruction of the electroweak interactions takes place at the last stage ( $\approx \epsilon^4$ ) together with restoration of mass of all particles.

Two other generations of leptons and quarks are developed in a similar way: for infinite energy there are only massless right  $\mu$ - and  $\tau$ -muons, left  $\mu$ - and  $\tau$ -neutrinos, as well as

massless left and right quarks  $c_l, c_r, s_r, t_l, t_r, b_r$ .  $c$ - and  $t$ -quarks first acquire their mass and after that  $\mu$ -,  $\tau$ -muons,  $s$ -,  $b$ -quarks become massive.

### 3 QCD with contracted gauge group

Strong interactions of quarks are described by the QCD. Like the Electroweak Model QCD is a gauge theory based on the local color degrees of freedom [12]. The QCD gauge group is  $SU(3)$ , acting in three dimensional complex space  $\mathbf{C}_3$  of color quark states. The  $SU(3)$  gauge bosons are called gluons. There are eight gluons in total, which are the force carrier of the theory between quarks. The QCD Lagrangian is taken in the form

$$\mathcal{L} = \sum_q \bar{q}^i (i\gamma^\mu) (D_\mu)_{ij} q^j - \frac{1}{4} \sum_{\alpha=1}^8 F_{\mu\nu}^\alpha F^{\mu\nu\alpha}, \quad (17)$$

where  $D_\mu q$  are covariant derivatives of quark fields  $q = u, d, s, c, b, t$

$$D_\mu q = \left( \partial_\mu - ig_s \left( \frac{\lambda^\alpha}{2} \right) A_\mu^\alpha \right) q, \quad q = \begin{pmatrix} q_1 \\ q_2 \\ q_3 \end{pmatrix} \equiv \begin{pmatrix} q_R \\ q_G \\ q_B \end{pmatrix} \in \mathbf{C}_3, \quad (18)$$

$g_s$  is the strong coupling constant,  $t^a = \lambda^a/2$  are generators of  $SU(3)$ ,  $\lambda^a$  are Gell-Mann matrices in the form

$$\begin{aligned} \lambda^1 &= \begin{pmatrix} \cdot & 1 & \cdot \\ 1 & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}, \quad \lambda^2 = \begin{pmatrix} \cdot & -i & \cdot \\ i & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}, \quad \lambda^3 = \begin{pmatrix} 1 & \cdot & \cdot \\ \cdot & -1 & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}, \\ \lambda^4 &= \begin{pmatrix} \cdot & \cdot & 1 \\ \cdot & \cdot & \cdot \\ 1 & \cdot & \cdot \end{pmatrix}, \quad \lambda^5 = \begin{pmatrix} \cdot & \cdot & -i \\ \cdot & \cdot & \cdot \\ i & \cdot & \cdot \end{pmatrix}, \quad \lambda^6 = \begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & 1 \\ \cdot & 1 & \cdot \end{pmatrix}, \\ \lambda^7 &= \begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & -i \\ \cdot & i & \cdot \end{pmatrix}, \quad \lambda^8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & \cdot & \cdot \\ \cdot & 1 & \cdot \\ \cdot & \cdot & -2 \end{pmatrix}, \end{aligned} \quad (19)$$

gluon stress tensor

$$F_{\mu\nu}^\alpha = \partial_\mu A_\nu^\alpha - \partial_\nu A_\mu^\alpha + g_s f^{\alpha\beta\gamma} A_\mu^\beta A_\nu^\gamma, \quad (20)$$

with the nonzero antisymmetric on all indices structure constant of the gauge group:

$$\begin{aligned} f^{123} &= 1, \quad f^{147} = f^{246} = f^{257} = f^{345} = \frac{1}{2}, \\ f^{156} &= f^{367} = -\frac{1}{2}, \quad f^{458} = f^{678} = \frac{\sqrt{3}}{2}, \end{aligned} \quad (21)$$

where  $[t^\alpha, t^\beta] = if^{\alpha\beta\gamma} t^\gamma$ ,  $\alpha, \beta, \gamma = 1, \dots, 8$ . Mass terms  $-m_q \bar{q}^i q_i$  are not included as far as they are present in the electroweak Lagrangian.

The choice of Gell-Mann matrices in the form (19) fix the basis in  $SU(3)$ . This enable us to write out the covariant derivatives (18) in the explicit form

$$D_\mu = \mathbf{I}\partial_\mu - i\frac{g_s}{2} \begin{pmatrix} A_\mu^3 + \frac{1}{\sqrt{3}}A_\mu^8 & A_\mu^1 - iA_\mu^2 & A_\mu^4 - iA_\mu^5 \\ A_\mu^1 + iA_\mu^2 & \frac{1}{\sqrt{3}}A_\mu^8 - A_\mu^3 & A_\mu^6 - iA_\mu^7 \\ A_\mu^4 + iA_\mu^5 & A_\mu^6 + iA_\mu^7 & -\frac{2}{\sqrt{3}}A_\mu^8 \end{pmatrix} =$$

$$= \mathbf{I}\partial_\mu - i\frac{g_s}{2} \begin{pmatrix} A_\mu^{RR} & A_\mu^{RG} & A_\mu^{RB} \\ A_\mu^{GR} & A_\mu^{GG} & A_\mu^{GB} \\ A_\mu^{BR} & A_\mu^{BG} & A_\mu^{BB} \end{pmatrix}, \quad (22)$$

where

$$A_\mu^{RR} = \frac{1}{\sqrt{3}}A_\mu^8 + A_\mu^3, \quad A_\mu^{GG} = \frac{1}{\sqrt{3}}A_\mu^8 - A_\mu^3, \quad A_\mu^{BB} = -\frac{2}{\sqrt{3}}A_\mu^8,$$

$$A_\mu^{RR} + A_\mu^{GG} + A_\mu^{BB} = 0, \quad A_\mu^{GR} = A_\mu^1 + iA_\mu^2 = \bar{A}_\mu^{RG},$$

$$A_\mu^{BR} = A_\mu^4 + iA_\mu^5 = \bar{A}_\mu^{RB}, \quad A_\mu^{BG} = A_\mu^6 + iA_\mu^7 = \bar{A}_\mu^{GB}. \quad (23)$$

Let us note, that in QCD the special mechanism of spontaneous symmetry breaking is absent, therefore gluons are massless particles.

The contracted special unitary group  $SU(3; \kappa)$  is defined by the action

$$q'(\kappa) = \begin{pmatrix} q'_1 \\ \kappa_1 q'_2 \\ \kappa_1 \kappa_2 q'_3 \end{pmatrix} = \begin{pmatrix} u_{11} & \kappa_1 u_{12} & \kappa_1 \kappa_2 u_{13} \\ \kappa_1 u_{21} & u_{22} & \kappa_2 u_{23} \\ \kappa_1 \kappa_2 u_{31} & \kappa_2 u_{32} & u_{33} \end{pmatrix} \begin{pmatrix} q_1 \\ \kappa_1 q_2 \\ \kappa_1 \kappa_2 q_3 \end{pmatrix} =$$

$$= U(\kappa)q(\kappa), \quad \det U(\kappa) = 1, \quad U(\kappa)U^\dagger(\kappa) = 1 \quad (24)$$

on the complex space  $\mathbf{C}_3(\kappa)$  in such a way that the hermitian form

$$q^\dagger(\kappa)q(\kappa) = |q_1|^2 + \kappa_1^2 (|q_2|^2 + \kappa_2^2 |q_3|^2) \quad (25)$$

remains invariant, when the contraction parameters tend to zero:  $\kappa_1, \kappa_2 \rightarrow 0$ . Transition from the classical group  $SU(3)$  and space  $\mathbf{C}_3$  to the group  $SU(3; \kappa)$  and space  $\mathbf{C}_3(\kappa)$  is given by the substitution

$$q_1 \rightarrow q_1, \quad q_2 \rightarrow \kappa_1 q_2, \quad q_3 \rightarrow \kappa_1 \kappa_2 q_3,$$

$$A_\mu^{GR} \rightarrow \kappa_1 A_\mu^{GR}, \quad A_\mu^{BG} \rightarrow \kappa_2 A_\mu^{BG}, \quad A_\mu^{BR} \rightarrow \kappa_1 \kappa_2 A_\mu^{BR}, \quad (26)$$

and diagonal gauge fields  $A_\mu^{RR}, A_\mu^{GG}, A_\mu^{BB}$  remain unchanged.

Substituting (26) in (17), we obtain the quark part of Lagrangian in the form

$$\mathcal{L}_q(\kappa) = \sum_q \left\{ i\bar{q}_1 \gamma^\mu \partial_\mu q_1 + \frac{g_s}{2} |q_1|^2 \gamma^\mu A_\mu^{RR} + \right.$$

$$+ \kappa_1^2 \left[ i\bar{q}_2 \gamma^\mu \partial_\mu q_2 + \frac{g_s}{2} (|q_2|^2 \gamma^\mu A_\mu^{GG} + q_1 \bar{q}_2 \gamma^\mu A_\mu^{GR} + \bar{q}_1 q_2 \gamma^\mu \bar{A}_\mu^{GR}) \right] +$$

$$+ \kappa_1^2 \kappa_2^2 \left[ i\bar{q}_3 \gamma^\mu \partial_\mu q_3 + \frac{g_s}{2} (|q_3|^2 \gamma^\mu A_\mu^{BB} + q_1 \bar{q}_3 \gamma^\mu A_\mu^{BR} + \bar{q}_1 q_3 \gamma^\mu \bar{A}_\mu^{BR} + \right.$$

$$\left. + q_2 \bar{q}_3 \gamma^\mu A_\mu^{BG} + \bar{q}_2 q_3 \gamma^\mu \bar{A}_\mu^{BG} \right) \Big] \Big\} = L_q^\infty + \kappa_1^2 L_q^{(2)} + \kappa_1^2 \kappa_2^2 L_q^{(4)}. \quad (27)$$

Let us introduce the notations

$$\partial A^k \equiv \partial_\mu A_\nu^k - \partial_\nu A_\mu^k, \quad [k, m] \equiv A_\mu^k A_\nu^m - A_\mu^m A_\nu^k, \quad (28)$$

then the gluon tensor has the following components

$$\begin{aligned} F_{\mu\nu}^1 &= \kappa_1 \left\{ \partial A^1 + \frac{g_s}{2} \left( 2[2, 3] + \kappa_2^2 ([4, 7] - [5, 6]) \right) \right\}, \\ F_{\mu\nu}^2 &= \kappa_1 \left\{ \partial A^2 + \frac{g_s}{2} \left( -2[1, 3] + \kappa_2^2 ([4, 6] + [5, 7]) \right) \right\}, \\ F_{\mu\nu}^3 &= \partial A^3 + \frac{g_s}{2} \left( \kappa_1^2 2[1, 2] - \kappa_2^2 [6, 7] + \kappa_1^2 \kappa_2^2 [4, 5] \right), \\ F_{\mu\nu}^4 &= \kappa_1 \kappa_2 \left\{ \partial A^4 - \frac{g_s}{2} \left( [1, 7] + [2, 6] + [3, 5] - \sqrt{3}[5, 8] \right) \right\}, \\ F_{\mu\nu}^5 &= \kappa_1 \kappa_2 \left\{ \partial A^5 + \frac{g_s}{2} \left( [1, 6] - [2, 7] + [3, 4] - \sqrt{3}[4, 8] \right) \right\}, \\ F_{\mu\nu}^6 &= \kappa_2 \left\{ \partial A^6 + \frac{g_s}{2} \left( \kappa_1^2 ([2, 4] - [1, 5]) + [3, 7] + \sqrt{3}[7, 8] \right) \right\}, \\ F_{\mu\nu}^7 &= \kappa_2 \left\{ \partial A^7 + \frac{g_s}{2} \left( \kappa_1^2 ([1, 4] + [2, 5]) - [3, 6] - \sqrt{3}[6, 8] \right) \right\}, \\ F_{\mu\nu}^8 &= \partial A^8 + \frac{g_s \sqrt{3}}{2} \kappa_2^2 \left( \kappa_1^2 [4, 5] + [6, 7] \right). \end{aligned} \quad (29)$$

The gluon part of Lagrangian is as follows

$$\begin{aligned} \mathcal{L}_{gl}(\kappa) &= -\frac{1}{4} F_{\mu\nu}^\alpha F^{\mu\nu \alpha} = \\ &= -\frac{1}{4} \left\{ H_3^2 + H_8^2 + \kappa_1^2 (F_1^2 + F_2^2 + 2H_3 F_3) + \right. \\ &\quad + \kappa_2^2 (G_6^2 + G_7^2 + 2H_3 G_3 - 2\sqrt{3} H_8 G_3) + \kappa_1^4 F_3^2 + \kappa_2^4 4G_3^2 + \\ &\quad + \kappa_1^2 \kappa_2^2 [P_4^2 + P_5^2 + 2(F_1 G_1 + F_2 G_2 + F_3 G_3 + F_6 G_6 + F_7 G_7 + \\ &\quad + \sqrt{3} H_8 P_3)] + \kappa_1^2 \kappa_2^4 (G_1^2 + G_2^2 - 4G_3 P_3) + \\ &\quad \left. + \kappa_1^4 \kappa_2^2 (F_6^2 + F_7^2 + 2F_3 P_3) + \kappa_1^4 \kappa_2^4 4P_3^2 \right\}. \end{aligned} \quad (30)$$

where

$$\begin{aligned} F_1 &= \partial A^1 + g_s [2, 3], \quad F_2 = \partial A^2 - g_s [1, 3], \\ G_1 &= \frac{g_s}{2} ([4, 7] - [5, 6]), \quad G_2 = \frac{g_s}{2} ([4, 6] + [5, 7]), \end{aligned}$$



$$\begin{aligned}
H_3 &= \partial A^3, \quad F_3 = g_s[1, 2], \quad G_3 = -\frac{g_s}{2}[6, 7], \quad P_3 = \frac{g_s}{2}[4, 5], \\
P_4 &= \partial A^4 - \frac{g_s}{2}([1, 7] + [2, 6] + [3, 5] - \sqrt{3}[5, 8]), \\
P_5 &= \partial A^5 + \frac{g_s}{2}([1, 6] - [2, 7] + [3, 4] - \sqrt{3}[4, 8]), \\
G_6 &= \partial A^6 + \frac{g_s}{2}([3, 7] + \sqrt{3}[7, 8]), \quad F_6 = \frac{g_s}{2}([2, 4] - [1, 5]), \\
G_7 &= \partial A^7 - \frac{g_s}{2}([3, 6] + \sqrt{3}[6, 8]), \quad F_7 = \frac{g_s}{2}([1, 4] + [2, 5]), \\
H_8 &= \partial A^8.
\end{aligned} \tag{31}$$

In the framework of Cayley-Klein scheme [2] the gauge group  $SU(3; \kappa)$  has two one-parameter contractions  $\kappa_1 \rightarrow 0, \kappa_2 = 1$  and  $\kappa_2 \rightarrow 0, \kappa_1 = 1$ , as well as one two-parameter contraction  $\kappa_1, \kappa_2 \rightarrow 0$ . We consider the following contraction:  $\kappa_1 = \kappa_2 = \kappa = \epsilon \rightarrow 0$ , which corresponds to the infinite temperature limit of QCD. The quark part of Lagrangian (27) is represented as a sum of terms proportional to zero, the second and forth powers of contraction parameter  $\epsilon$

$$\mathcal{L}_q(\epsilon) = L_q^\infty + \epsilon^2 L_q^{(2)} + \epsilon^4 L_q^{(4)} \tag{32}$$

and gluon part is represented as a sum

$$\mathcal{L}_{gl}(\epsilon) = L_{gl}^\infty + \epsilon^2 L_{gl}^{(2)} + \epsilon^4 L_{gl}^{(4)} + \epsilon^6 L_{gl}^{(6)} + \epsilon^8 L_{gl}^{(8)}. \tag{33}$$

In the infinite temperature limit  $\kappa = \epsilon \rightarrow 0$  the most parts of gluon tensor components are equal to zero and the expressions for two nonzero components are simplified

$$F_{\mu\nu}^3 = \partial_\mu A_\nu^3 - \partial_\nu A_\mu^3 = \frac{1}{2}(F_{\mu\nu}^{RR} - F_{\mu\nu}^{GG}), \quad F_{\mu\nu}^8 = \partial_\mu A_\nu^8 - \partial_\nu A_\mu^8 = \frac{\sqrt{3}}{2}(F_{\mu\nu}^{RR} + F_{\mu\nu}^{GG}), \tag{34}$$

so we can write out the QCD Lagrangian  $\mathcal{L}_\infty$  in this limit explicitly

$$\begin{aligned}
\mathcal{L}_\infty &= L_q^\infty + L_{gl}^\infty = \\
&= \sum_q \left\{ i \bar{q}_R \gamma^\mu \partial_\mu q_R + \frac{g_s}{2} |q_R|^2 \gamma^\mu A_\mu^{RR} \right\} - \frac{1}{4} (F_{\mu\nu}^{RR})^2 - \frac{1}{4} (F_{\mu\nu}^{GG})^2 - \frac{1}{4} F_{\mu\nu}^{RR} F_{\mu\nu}^{GG}.
\end{aligned} \tag{35}$$

From  $\mathcal{L}_\infty$  we conclude that only the dynamic terms for the first color component of massless quarks survive under infinite temperature, which means that the quarks are monochromatic, and the terms also survive, which describe the interactions of these components with  $R$ -gluons. Besides  $R$ -gluons there are also  $G$ -gluons, which do not interact with the quarks.

Similarly to the Electroweak Model starting with  $\mathcal{L}_q(\epsilon)$  (32) and  $\mathcal{L}_{gl}(\epsilon)$  (33) one can construct a number of intermediate models for QCD, which describe the gradual restoration of color degrees of freedom for the quarks and the gluon interactions in the Universe evolution.

It follows from Lagrangian  $\mathcal{L}(\epsilon) = \mathcal{L}_q(\epsilon) + \mathcal{L}_{gl}(\epsilon)$ , that the total reconstruction of the quark color degrees of freedom will take place after the restoration of all quark mass ( $\approx \epsilon^2$ ) at the same time with the reestablishment of all electroweak interactions ( $\approx \epsilon^4$ ). Complete color interactions start to work later because some of them are proportionate to the eighth power  $\epsilon^8$ .

## 4 Estimation of boundary values

As it was mentioned the contraction of gauge group of QCD gives an opportunity to order in time different stages of its development, but does not make it possible to bear their absolute date. Let us try to estimate this date with the help of additional assumptions. The equality of the contraction parameters for QCD and the EWM is one of these assumptions.

Then we use the fact that the electroweak epoch starts at the temperature  $T_4 = 100 \text{ GeV}$  ( $1 \text{ GeV} = 10^{13} \text{ K}$ ) and the QCD epoch begins at  $T_8 = 0,2 \text{ GeV}$ . In other words we assume that complete reconstruction of the EWM, whose Lagrangian has minimal terms proportionate to  $\epsilon^4$ , and QCD, whose Lagrangian has minimal terms proportionate to  $\epsilon^8$ , take place at these temperatures. Let us denote by  $\Delta$  cutoff level for  $\epsilon^k$ ,  $k = 1, 2, 4, 6, 8$ , i.e. for  $\epsilon^k < \Delta$  all the terms proportionate to  $\epsilon^k$  are negligible quantities in Lagrangian. At last we suppose that the contraction parameter inversely depends on temperature

$$\epsilon(T) = \frac{A}{T}, \quad (36)$$

where  $A$  is constant.

As far as the minimal terms in the QCD Lagrangian are proportional to  $\epsilon^8$  and QCD is completely reconstructed at  $T_8 = 0,2 \text{ GeV}$ , we have the equation  $\epsilon^8(T_8) = A^8 T_8^{-8} = \Delta$  and obtain  $A = T_8 \Delta^{1/8} = 0,2 \Delta^{1/8} \text{ GeV}$ . The minimal terms in the EWM Lagrangian are proportional to  $\epsilon^4$  and it is reconstructed at  $T_4 = 100 \text{ GeV}$ , so we have  $\epsilon^4(T_4) = A^4 T_4^{-4} = \Delta$ , i.e.  $T_4 = A \Delta^{-1/4} = T_8 \Delta^{1/8} \Delta^{-1/4} = T_8 \Delta^{-1/8}$  and we obtain the cutoff level  $\Delta = (T_8 T_4^{-1})^8 = (0,2 \cdot 10^{-2})^8 \approx 10^{-22}$ , which is consistent with the typical energies of the Standard Model. From the equation  $\epsilon^k(T_k) = A^k T_k^{-k} = \Delta$  we obtain

$$T_k = \frac{A}{\Delta^{1/k}} = \frac{T_8 \Delta^{1/8}}{\Delta^{1/k}} = T_8 \Delta^{\frac{k-8}{8k}} \approx 10^{\frac{88-15k}{4k}} \text{ GeV}. \quad (37)$$

Simple calculations give the following estimations for the boundary values of the temperature in the early Universe ( $\text{GeV}$ ):  $T_1 = 10^{18}$ ,  $T_2 = 10^7$ ,  $T_3 = 10^3$ ,  $T_4 = 10^2$ ,  $T_6 = 1$ ,  $T_8 = 2 \cdot 10^{-1}$ . The obtained estimation for "infinity" temperature  $T_1 \approx 10^{18} \text{ GeV}$  is comparable with Planck energy  $\approx 10^{19} \text{ GeV}$ , where the gravitation effects are important. So the developed evolution of the elementary particles does not exceed the range of the problems described by electroweak and strong interactions.

It should be noted that for the power function class  $\epsilon(T) = BT^{-p}$  the estimations for temperature boundary values are very weakly dependent on power  $p$ . So for  $p = 10$  we obtain practically the same  $T_k$  as for the simplest function (36) with  $p = 1$ .

## Conclusion

We have investigated the high-temperature limit of the SM which was obtained from the first principles of the gauge theory as contraction of its gauge group. It was shown that the mathematical contraction parameter is proportional to the temperature and its zero limit corresponds to the infinite temperature limit of the Model. The SM passes in this limit through several stages, which are distinguished by the powers of contraction parameter, what gives the opportunity to classify them in time as earlier-later. To determine the absolute date of these stages the additional assumptions was used, namely: the inversely

dependence  $\epsilon$  on the temperature (36) and the cutoff level  $\Delta$  for  $\epsilon^k$ . Unknown parameters are determined with the help of the QCD and EWM typical energies.

The exact expressions for the respective Lagrangians for any stage in the SM evolution are presented. On the base of decompositions (13), (32), (33) the intermediate models  $\mathcal{L}_k$  for any temperature scale are constructed. It gives an opportunity to draw a conclusions on interactions and properties of elementary particles in each of considered epoch. The presence of the several intermediate models in the interval from Plank energy  $10^{19} \text{ GeV}$  up to the EWM typical energy  $10^2 \text{ GeV}$  instead of only one model automatically takes away so-called hierarchy problem of the SM [12].

At the infinite temperature limit ( $T > 10^{18} \text{ GeV}$ ) all particles including vector bosons lose their masses and electroweak interactions become long-range. Monochromatic massless quarks exchange by only one sort of  $R$ -gluons. Besides  $R$ -gluons there are also  $G$ -gluons, which do not interact with quarks. It follows from the explicit form of Lagrangians  $L_\infty^{int}(A_\mu, Z_\mu)$  (15) and  $\mathcal{L}_\infty$  (35) that only the particles of the same sort interact with each other. Particles of different sorts do not interact. It looks like some stratification of leptons and quark-gluon plasma with only one sort of particles in each stratum.

At the level of classical gauge fields it is already possible to give some conclusions on the appearance of elementary particles mass on the different stages of the Universe evolution. In particular we can conclude that half of quarks ( $\approx \epsilon$ ,  $10^{18} \text{ GeV} > T > 10^7 \text{ GeV}$ ) first restore they mass. Then  $Z$ -bosons, electrons and other quarks become massive ( $\approx \epsilon^2$ ,  $10^7 \text{ GeV} > T > 10^3 \text{ GeV}$ ). Finally Higgs boson  $\chi$  and charged  $W^\pm$ -bosons restore their masses because their mass terms are multiplied by  $\epsilon^4$  ( $T < 10^2 \text{ GeV}$ ).

In a similar way it is possible to describe the evolution of particle interactions. Self-action of Higgs boson appears with its mass restoration. At the same epoch start interactions of four  $W^\pm$ -bosons, as well as of two Higgs and two  $W$ -bosons (7). The only one term in Lagrangian, which is proportional to the third power of  $\epsilon$ , describes interaction of Higgs boson with charged  $W^\pm$ -bosons ( $T < 10^3 \text{ GeV}$ ). The rest of the electroweak particle interactions are appeared in the second order of the contraction parameter ( $10^7 \text{ GeV} > T > 10^3 \text{ GeV}$ ). Some part of color interactions between quarks in Lagrangian (27) is proportional to  $\epsilon^2$  ( $T < 10^7 \text{ GeV}$ ) and the rest part is proportional to  $\epsilon^4$  ( $T < 10^2 \text{ GeV}$ ). Therefore the complete restoration of quark color degrees of freedom takes place after the appearance of quark masses ( $\approx \epsilon^2$ ,  $T < 10^7 \text{ GeV}$ ) (16) together with the restoration of all electroweak interactions ( $\approx \epsilon^4$ ,  $T < 10^2 \text{ GeV}$ ). Complete color interactions start later because they are proportional to  $\epsilon^8$  ( $T < 10^{-1} \text{ GeV}$ ).

The evolution of elementary particles and their interactions in the early Universe obtained with the help of contractions of gauge groups of the SM does not contradict the canonical one [6], according to which the QCD phase transitions take place later then the electroweak phase transitions. The developed evolution of the SM present the basis for a more detailed analysis of different phases in the formation of leptons and quark-gluon plasma.

The author is thankful to V. V. Kuratov and V. I. Kostyakov for helpful discussions. The study is supported by Program of UrD RAS project N 15-16-1-3.

## References

- [1] E. İnönü and E.P. Wigner, *On the contraction of groups and their representations*, *Proc. Nat. Acad. Sci. USA* **39** (1953) 510.
- [2] N.A. Gromov, *Contractions of Classical and Quantum Groups*. Moscow: Fizmatlit, 2012. 318 p. [in Russian].
- [3] N.A. Gromov, *Contraction of Electroweak Model and neutrino*, *Physics of Atomic Nuclei* **75** (2012) 1203.
- [4] N.A. Gromov, *Interpretation of neutrino-matter interactions at low energies as contraction of gauge group of Electroweak Model*, *Physics of Atomic Nuclei* **76** (2013) 1144.
- [5] N.Yu. Reshetikhin, L.A. Takhtajan and L.D. Faddeev, *Quantization of Lie groups and Lie algebras*, *Algebra i Analiz* **1** (1989) 178 [in Russian].
- [6] D.S. Gorbunov and V.A. Rubakov, *Introduction to the Theory of the Early Universe: Hot Big Bang Theory*. Singapore: World Scientific, 2011. 488 p.
- [7] N.A. Gromov, *High and low energy limits of Electroweak Model*, *Proc. Komi SC UrD RAS* **1(17)** (2014) 5 [in Russian].
- [8] N.A. Gromov, *Natural Limits of Electroweak Model as Contraction of its Gauge Group*, *Physica Scripta* **90** (2015) 074009.
- [9] V.A. Rubakov *Classical Theory of Gauge Fields*. Princeton, USA: University Press, 2002. 444 p.
- [10] M.E. Peskin and D.V. Schroeder, *An Introduction to Quantum Field Theory*. Addison-Wesley, 1995.
- [11] L.D. Linde, *Particle Physics and Inflationary Cosmology*. Moscow: Nauka, 1990. 280 p. [in Russian].
- [12] V.M. Emel'yanov, *Standard Model and its expansion*. Moscow: Fizmatlit, 1990. 584 p. [in Russian].